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Magnetic scattering and unusual behavior of the critical fields and other parameters: Application to borocarbides

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Abstract. The pair-breaking effect and its impact on the properties of borocarbides is studied. The pair-breaking effect caused by localized magnetic moments drastically affects the superconducting properties. Interaction between the magnetic moments and the resulting ordering trend lead to a behavior entirely different from the conventional picture. The main focus is on the behavior of the upper (H_{c2}) and lower (H_{c1}) critical fields. In addition, the temperature dependence of several quantities (penetration depth, coherence length, etc.) has been calculated. The theory has been applied to the borocarbide $LuNi_2B_2C$ and is in very good agreement with the recent experimental data.

PACS. 74.25.Ha Magnetic properties – 74.70.Dd Ternary, quaternary and multinary compounds (including Chevrel phases, borocarbides etc.) – 74.62.Dh Effects of crystal defects, doping and substitution

1 Introduction

This paper is concerned with unusual properties of the borocarbides. Borocarbides are remarkable materials in which there is an interplay between superconductivity and magnetism (see *e.g.* [1–15]). Our study was particularly motivated by recent experimental data [15] in which unconventional temperature dependencies $H_{c1}(T)$ and $H_{c2}(T)$ for $LuNi_2B_2C$ (see Fig. 1) drastically different from those for conventional superconductors [16] have been observed. We paid a special attention to the low temperature region where the almost linear temperature dependence of the critical field has been observed. Moreover, the paper [15] contains experimental data not only on H_{c2}, but also on the lower critical field H_{c1}. It was motivated us to develop a rigorous microscopic theory describing the temperature dependence $H_{c1}(T)$ in a whole temperature range (in the presence of pair-breaking as well as in the absence of magnetic scattering), not only near T_c. This derivation (see Sect. 2) is interesting for its own sake.

The unconventional behavior of the upper critical field observed in the overdoped cuprates [17,18] as well as in the YZnCuO compound [19]. We studied this problem in our papers [20–22] and concluded that this effect is due to magnetic scattering by pair-breakers (magnetic impurities) and of their ordering trend. We have shown also in [20] that the presence of inhomogeneities leads to unusual curvature of H_{c2} near T_c. We expect that borocarbide behavior also can be described by our theory, although the properties and parameters of these materials are very different from those of the cuprates. As a result, the dependence $H_{c2}(T)$ for $LuNi_2B_2C$ differs from that observed in [17–19], but is also non-conventional. The almost linear temperature dependence of H_{c2} is spread over large temperature range (see below, Fig. 1). It is essential that the borocarbides, like $LuNi_2B_2C$, contain magnetic impurities and their presence has been observed experimentally (see below, Sect. 3). In addition, we focus on the problem of $H_{c1}(T)$; this dependence was measured for the borocarbides in [15].

The structure of the paper is as follows. The theoretical approach is described in Section 2. An analysis of the experimental data is given in Section 3.

2 Theory

2.1 Main equations. Upper critical field

Our approach is based on the method of integrated Green's functions [23,24]. We consider the "dirty" anisotropic case. The basic equations have the following form [25]

$$\alpha\Delta-\tilde{\beta}\omega+0.5\hat{D}_{ij}\left[\alpha(\partial_-)_i(\partial_-)_j\tilde{\beta}-\tilde{\beta}\left(\frac{\partial^2\alpha}{\partial r_i\partial r_j}\right)\right]=\alpha\tilde{\beta}\Gamma \tag{1a}$$

$$\Delta=2\pi T|\lambda|\sum_{\omega_n>0}\tilde{\beta},\quad \alpha^2+|\tilde{\beta}|^2=1 \tag{1b}$$

$$\mathbf{j}=-\mathrm{i}e\nu\hat{D}2\pi T\sum_{\omega_n>0}(\tilde{\beta}^*\partial_-\tilde{\beta}-\tilde{\beta}\partial_+\tilde{\beta}^*). \tag{1c}$$

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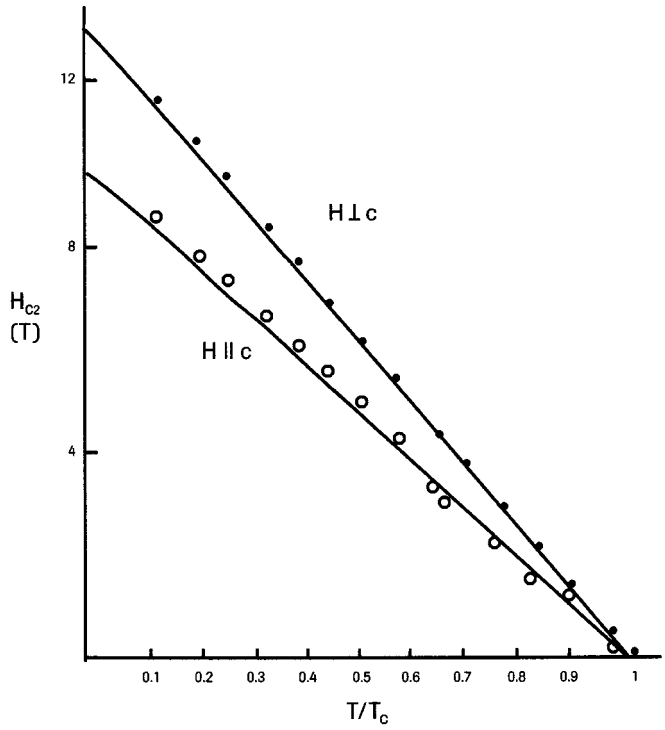

Fig. 1. The upper critical magnetic field H_{c2} for two orientations of the field: (•) experimental data [5], (—) theory.

Here α and $\tilde{\beta}$ are the usual and pairing Green's functions integrated over energy, Δ is the order parameter, $\omega_n = (2n+1)\pi T$, $\partial_\pm = (\partial/\partial\mathbf{r}) \pm 2ie\mathbf{A}$, $\mathbf{A}$ is the vector potential, ν is the density of states, and λ is the coupling constant; Γ is the magnetic scattering amplitude ($\Gamma = \tau_s^{-1}$, where τ_s is the spin-flip relaxation time [26]), and $\hat{D}_{ij}$ is the diffusion coefficient tensor.

We consider a compound which is anisotropic in one direction (this is appropriate to the borocarbides as well as to the cuprates; the c axis is chosen to lie along this direction). For concreteness, consider cylindrical sample with its axis aligned with the c-direction. In this case the tensor D_{ij} contains two different components $D_\parallel$ and $D_\perp$.

Based on the system (1) one can obtain the following equation describing the upper critical field H_{c2} (see [20]):

$$\ln\left(\frac{T_c}{T}\right) + \Psi\left(\frac{1}{2} + \frac{\Gamma(T_c)}{2\pi T_c}\right) - \Psi\left(\frac{1}{2} + \frac{\Gamma(T)}{2\pi T} + \frac{eH_{c2}D_\perp}{2\pi T}\right) = 0. \quad (2)$$

The magnetic scattering amplitude Γ is temperature dependent, and this dependence is a key ingredient in our approach. Note that according to the conventional theory [26,27], Γ does not depend on the temperature, which corresponds to the picture of independent localized magnetic moments. The presence of a $\Gamma(T)$ dependence which is stated explicitly in equation (2) reflects a correlation between the magnetic moments and, consequently, the frustration of spin-flip scattering as $T \to 0$. According to [20], the dependence $\Gamma(T)$ has a form

$$\Gamma(T) = \Gamma_0 f(\tau)$$
$$f(\tau) = (1 + \beta\tau)/(1 + \tau). \quad (3)$$

Here $\tau = T/\theta$, where θ is the characteristic temperature for the ordering trend of the magnetic moments. If a magnetic transition is observed at some temperature T^M, then $\theta = T^M$. For example, $\theta = 2$ K for the $Sm_{1.85}Ce_{0.15}CuO_{4-y}$ compound studied in [28] (this compound displays an antiferromagnetic transition due to the ordering of Sm^+ ions at $T^M = 2$ K; the superconducting transition takes place at $T_c = 9.5$ K). One should stress that while the correlation between magnetic moments does not necessarily imply the existence of an ordering transition, the accompanying frustration will, nevertheless, result in a temperature dependence of the parameter Γ. For example, such is the case with the overdoped Tl- and Bi-based cuprates. Here θ should be considered as a parameter manifested in the behavior of the critical field. For example, H_{c2} for the $Tl_2B_2CuO_8$ displays a sharp upturn near $T \cong 2$ K [17] (a more detailed analysis [20] has resulted in the close value $\theta = 1.6$ K for this compound). Another parameter which enters equation (3) is β which reflects the relative change in the magnetic scattering amplitude Γ caused by correlations. Indeed, correlation between the magnetic moments frustrates the spin-flip process, resulting in an increase in the spin-flip relaxation time τ_s and in a decrease in $\Gamma = \tau_s^{-1}$. The smallest value of the amplitude corresponds to $T = 0$ K; according to equation (3), it is equal to Γ_0. Let us denote by Γ_α the value of Γ in the absence of the correlation effect (this corresponds to $\theta = 0$). Then $\beta = \Gamma_\alpha/\Gamma_0$ and therefore the parameter β indeed describes the relative change in Γ caused by correlations. This is an adjustable parameter in our theory.

Therefore, the upper critical field is determined by equation (2) with $\Gamma(T)$ described by equation (3). Note that if the magnetic field is perpendicular to the c-axis, then $D_\perp$ should be replaced by the quantity $(D_\perp D_\parallel)^{1/2}$. The analysis of experimental data [15] will be described below (Sect. 3).

2.2 Order parameter. Penetration depth

Let us investigate the impact of magnetic scattering and its temperature dependence (Eq. (3)) on the major parameters of the system. We begin with the penetration depth.

Equations (1) can be written in the form:

$$-\left\{\rho^{-1}\partial/\partial\rho(\rho\partial/\partial\rho) - \rho^{-2}\right\}Q = 16\pi^2 e\nu D_\perp(\rho^{-1} - 2eQ)T\sum_{\omega>0}\cos^2\theta$$

$$|\Delta|\sin\theta - \omega\cos\theta - 0.5D_\perp\rho^{-1}\partial/\partial\rho(\rho\partial\theta/\partial\rho) = 0.5\sin(2\theta)\left(\Gamma + \left(\rho^{-1} - 2eQ\right)^2\right). \quad (4)$$

We used the following notation

$$\Delta = \Delta(\rho)\exp(\mathrm{i}\varphi); \quad \beta = \beta(\rho)\exp(\mathrm{i}\varphi);$$
$$\beta(\rho) = \cos\theta; \quad \alpha = \sin\theta; \quad \mathbf{A} = Q(\rho)\mathbf{e}_\varphi; \tag{5}$$

$\mathbf{e}_\varphi$ is an unit vector: $\mathbf{e}_\varphi = (-\sin\varphi; \cos\varphi)$. This notation is convenient for the case of cylindrical symmetry. Equations (4) follow after substitution of (5) into equations (1).

It is essential that equations (1) contain the order parameter Δ. The presence of magnetic scattering leads to this parameter being different from the energy gap [26,27], see below, equation (17).

For the large value of the Ginzburg-Landau (GL) parameter $\kappa = \lambda_\mathrm{L}/\xi$ (λ_L is the penetration depth and ξ is the coherence length) we obtain from equation (4)

$$2eQ = \rho^{-1} - \lambda_\mathrm{L}^{-1} K_1(\rho/\lambda_\mathrm{L}) \tag{6}$$

where K_1 is the Bessel function, and the penetration length λ_L is written as

$$\lambda_\mathrm{L}^{-2} = 32e^2\nu D_\perp \pi^2 T \sum_{\omega>0} \cos^2\theta. \tag{7}$$

Note also that based on equation (1b), we can obtain the following relation:

$$\alpha^2 \delta\Delta = \left\{ (\alpha^2 - \beta^2)\Gamma + \alpha\omega + \beta\Delta - \frac{D_\perp}{2}\frac{\partial^2}{\partial r^2} \right\} \delta\beta$$

which can be reduced to the form

$$2\pi T \sum_{\omega>0} \Big\{ \sin^2\theta \Big[-\Gamma\cos 2\theta + \omega\sin\theta + \Delta\cos\theta - \frac{D_\perp}{2\xi^2}\Big]^{-1} - \cos\theta/\Delta \Big\} = 0. \tag{8}$$

Equation (8) allows us to evaluate $\xi(T)$. After some manipulations (see Appendix) we obtain:

$$2\pi T \sum_{\omega>0} \left\{ \frac{\cos^2\theta(Z - \cos\theta + (\Gamma/\Delta)\cos^2\theta)}{1 - Z\cos\theta - (\Gamma/\Delta)\cos^3\theta} \right\} = 0 \tag{9}$$

where $Z = D_\perp/2\xi^2\Delta$.

Now we can evaluate the penetration depth defined by equation (7). Consider the limiting cases. In the low temperature region one can replace the sum by the integral over ω. As a result, we obtain at $T \to 0$

$$\lambda_\mathrm{L}^{-2} = 16\pi e^2 \nu D_\perp \left\{ \left(\frac{\pi}{2} - \theta(\Gamma)\right)\Delta - \Gamma(2/3 - 3\sin\theta(\Gamma)/4 - \sin(3\theta(\Gamma))/12) \right\}. \tag{10}$$

For concreteness we consider the case $\mathbf{H} \parallel c$. The function $\theta(\Gamma)$ has a form:

$$\theta(\Gamma) = \begin{cases} \arccos(\Delta/\Gamma) & (\Gamma > \Delta) \\ 0 & (\Gamma < \Delta) \end{cases}. \tag{11}$$

It is essential that the function $\lambda_\mathrm{L}(T)$ contains a linear section near $T = 0$ K. This is due to the linear temperature dependence of the amplitude Γ (Eq. (3)). The scale of this dependence is determined by specific values of the parameters α and β. Note that equation (10) contains $\Delta(T)$ which determined from equation (1b) and also is affected by the dependence $\Gamma(T)$ (see below, Sect. 3).

In the region near T_c one can use equations (7, 9) and we arrive at the expression:

$$\lambda_\mathrm{L}^{-2} = (8e^2\nu D_\perp/T)|\Delta|^2 \Psi'\left(0.5 + \frac{\Gamma(T)}{2\pi T}\right) \tag{12}$$

where $\Delta(T)$ is determined by equation (1b).

The behavior of $\lambda_\mathrm{L}(T)$ over the entire temperature range can be calculated numerically directly from equation (7) (see below, Sect. 3). The presence of the function $\Gamma(T)$ may lead to a deviation from the conventional BCS dependence.

2.3 Coherence length ξ(T). Ginzburg-Landau parameter κ

Equation (9) along with equation (1b) can be used in order to calculate $\xi(T)$ over the whole temperature range (see below, Sect. 3). If $T \to 0$, one can replace the sum in equation (9) by an integral. Near T_c one can use the relation

$$Z\Psi'\left(\frac{1}{2} + \frac{\Gamma}{2\pi T}\right) = \frac{\Delta}{2\pi T}\sum_{n=0}^{\infty}\left(n + \frac{1}{2}\right)\left(n + \frac{1}{2} + \frac{\Gamma}{2\pi T}\right)^{-4}. \tag{13}$$

If we put $\Gamma = 0$, we obtain the usual expression: $Z = 7\zeta(3)\Delta/\pi^3 T$.

After evaluating the penetration depth λ_L and the coherence length ξ, we can calculate the GL parameter $\kappa = \lambda_\mathrm{L}/\xi$. Contrary to the usual case, this parameter turns out to be strongly temperature dependent. This dependence is determined by equations (7, 9). As above, consider the limiting cases, $T \to 0$ and $T \to T_\mathrm{c}$. In addition, it is interesting to investigate the dependence κ on n, the concentration of the localized magnetic moments. For example, at $T = 0$, $\kappa^2 \cong 1.5(8\pi^2 e^2 \nu D_\perp^2)^{-1}$ if $\Gamma = 0$, and $\kappa^2 = (48\pi^2 e^2 \nu D_\perp^2)^{-1}$ if $\Gamma \gg \Delta$, so that $\kappa(\Gamma = 0)/\kappa(\Gamma \gg \Delta)_{|T=0\,\mathrm{K}} = 1.7$.

Near T_c the parameter κ is described by the following equation which directly follows from equations (10, 13):

$$\kappa^2 = -(16\pi e^2 \nu D_\perp)^{-1}\left[\Psi''\left(\frac{1}{2} + \frac{\Gamma}{2\pi T}\right) + \frac{\Gamma}{6\pi T}\Psi'''\left(\frac{1}{2} + \frac{\Gamma}{2\pi T}\right)\right]\left[\Psi'\left(\frac{1}{2} + \frac{\Gamma}{2\pi T}\right)\right]^{-2}.$$

As a result, we obtain $\kappa(\Gamma = 0)/\kappa(\Gamma \gg T_\mathrm{c}) = 0.48$.

As was noted above, the dependence $\kappa(T)$ over the whole temperature range follows from equations (7, 9) (see below, Sect. 3)

2.4 Critical field H_{c1}

We now turn to the analysis of the dependence of H_{c1} on T. This problem requires an additional effort. Indeed, it has been possible to evaluate the dependence $H_{c2}(T)$ over the whole temperature range [16] because the order parameter is always small near H_{c2}. The situation with H_{c1} is different, and the rigorous derivation exists only near T_c and is based on the Ginzburg-Landau equations, although there are some qualitative estimates (see, *e.g.* [29]). Nevertheless, one can show (see below) that, based on equation (1), one can obtain a rigorous expression for $H_{c1}(T)$ valid over a broad temperature region in the limit of large $\kappa = \lambda_L/\xi$. This is precisely the case for the borocarbides (see Sect. 3). We will start with this evaluation, which is interesting in its own right.

In order to evaluate the low critical field H_{c1} it is necessary to calculate the energy of single vortex. Let us write the expression for the free energy:

$$F^S - F^N = \nu \int d\mathbf{r} \left\{ |\lambda|^{-1}|\Delta|^2 - 2\pi T \sum_{\omega>0} G \right\} + \frac{1}{8\pi} \int d\mathbf{r} \left[(\mathrm{rot}\,\mathbf{A})^2 - 2\mathbf{H}_0\,\mathrm{rot}\,\mathbf{A} \right] \quad (14)$$

where

$$\delta G/\delta|\Delta| = 2\cos\theta. \quad (14')$$

This is a general form of a free energy functional which leads to equations (1).

Based on equation (14), one can write the following expression which determines H_{c1}:

$$(H_{c1}/4\pi)\phi = \delta\tilde{F};$$
$$\delta\tilde{F} = \frac{1}{4}\int_0^\infty d\rho\,\rho H^2(\rho) + 2\pi\nu \int_0^\infty d\rho\,\rho\delta\{G\}. \quad (15)$$

Here $\phi = \pi/e$ is the flux quantum, and $\delta\{G\}$ is the energy difference given by the first term in equation (14) and caused by an appearance of an isolated vortex. This quantity can be calculated (see Appendix) and we arrive at the expression:

$$(H_{c1}/4\pi)\phi = (16e^2\lambda_L^2)^{-1}[\ln(\lambda_L/\xi) + \chi]. \quad (16)$$

Here $\chi = -C + \ln 2 + \gamma_1$, C is the Euler constant, and $\gamma_1 \leq 1$. Near T_c we obtain $\chi = 0.146$.

3 Experimental data: Discussion

The paper [15] contains a detailed experimental study of the borocarbide, $LuNi_2B_2C$. As was mentioned above, the measured upper and low critical fields display an unusual temperature dependence (see Figs. 1 and 4) drastically different from that of conventional superconductors. It is possible to show that the observed behavior can be explained by the magnetic scattering of correlated localized magnetic moments.

Let us note, at first, that the presence of localized magnetic moments in $LuNi_2B_2C$ has been established experimentally in [9,5]. In addition, the authors of [5] explicitly separated the contribution to the magnetic susceptibility in YNi_2B_2C caused by magnetic impurities.

Let us start with H_{c2}. The paper [15] contains data for longitudinal ($\mathbf{H} \parallel c$) and transverse ($\mathbf{H} \perp c$) upper critical fields for $LuNi_2B_2C$. Similar data were reported in [7]. The paper [15] contains also data for $H_{c1}(T)$. Note also that the unusual dependence $H_{c2}(T)$ for some borocarbides, *e.g.* for YNi_2B_2C compound (for $T > 4$ K), was observed in [7,10,14]. In this paper we analyze data for $LuNi_2B_2C$.

The temperature dependence of the critical fields can be evaluated with the use of equations (2, 16). The strong deviation of $H_{c2}(T)$ from the conventional dependence is due to the temperature dependence of the magnetic amplitude $\Gamma(T)$ (see Eq. (3)). Let us consider the case $H \parallel c$; first we can use the measured dependence $H_{c2}(T)$ in order to determine the major parameters. The parameters are $\theta = 33$ K, $\beta = 5.1$, $eD_\parallel = 1.45$ K/T. Furthermore, we find that $\Gamma_0 = 9$ K. It is interesting to note that, contrary to the cuprates (see [20]), the value of θ is larger than T_c. We think that this factor leads to the dependence $H_{c2}(T)$ for $LuNi_2B_2C$ (Fig. 1) being so different from those for the cuprates [17–19].

We now use the same values of β, θ and Γ_0 to evaluate H_{c2} for the case $H \perp c$. In order to obtain the theoretical curve, we need just one additional parameter $D_\perp$; its value is $eD_\perp = 2.5$ K/T. Again, one finds very good agreement with the data [15] (see Fig. 1).

Note that we focus mainly on the intermediate and low temperature region. As for the upward curvature near T_c, it can be affected, in addition to magnetic scattering (see the theoretical curve, Fig. 1), by inhomogeneity of the structure. Its impact is a two-fold. First of all, the presence of inhomogeneities directly affect the dependence $H_{c2}(T)$, see [20,21], and, in addition, it leads to smearing of the transition. As a result, the exact shape of the dependence $H_{c2}(T)$ near T_c is sensitive to the choice of the value of T_c (see discussion in [20]).

The paper [15] contains also the data on the low critical field H_{c1}. We discuss these data below. The dependence $H_{c1}(T)$ is determined (see Eq. (16)) by the temperature dependence of the parameter κ. Because of it, as the first step, we evaluate the temperature dependence of the order parameter $\Delta(T)$, the dependence $\lambda_L(T)$, the coherence length $\xi(T)$, and the GL parameter $\kappa(T)$ over the full temperature range. The calculation makes use of equations (1b, 7, 9). The results are presented in Figures 2 and 3. Clearly the dependence $\Delta(T)$ is very different from the usual BCS curve. This is due to the temperature dependence of Γ.

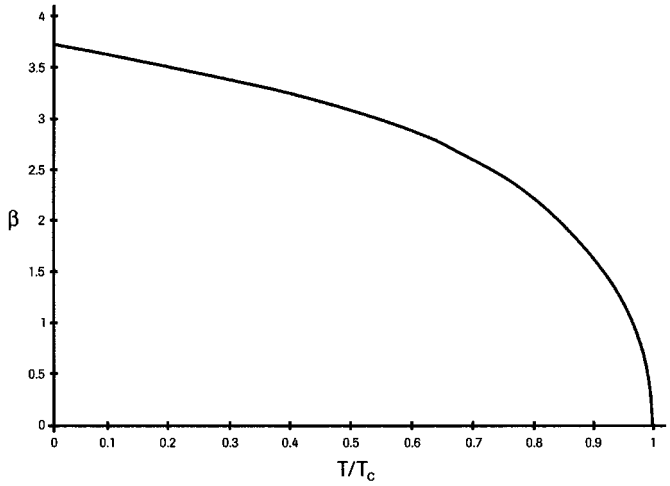

Fig. 2. Temperature dependence $\Delta(T)$ for $LuNi_2B_2C$; $\beta = \Delta/T_c$.

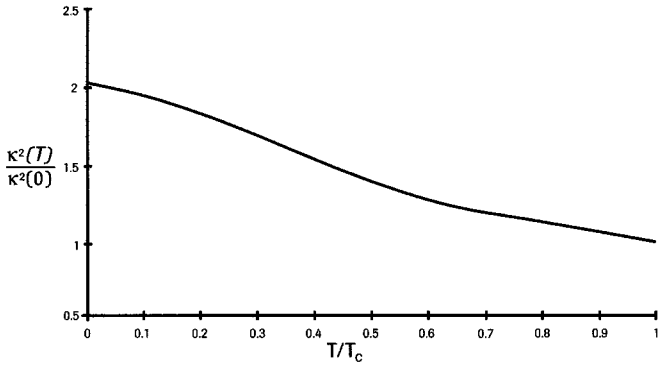

Fig. 3. Temperature dependence of the GL parameter κ.

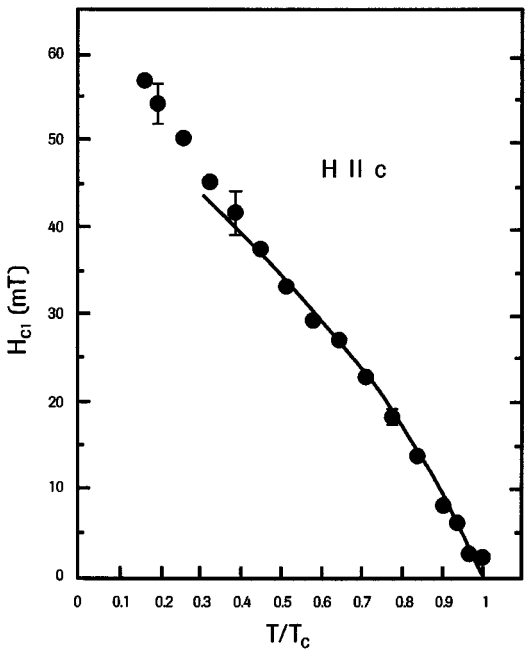

Fig. 4. The low critical field H_{c1}: (•) experimental data [15] for $LuNi_2B_2C$.

Note that the value $\Delta(0)/T_c$ appears to be much larger than that for the usual superconductors. This is caused by the effective decrease in the scattering amplitude $\Gamma(T)$ as $T \to 0$ K. This leads to an effective increase in the pairing ("recovery" effect, *cf.* [20]), and, correspondingly, to an increase in $\Delta(0)$.

As was mentioned above (see Sect. 2) the function $\Delta(T)$ is the order parameter, not the energy gap. For the usual BCS system $\Delta(T)$ is equal to the energy gap ε_0, but this is not the case for the pair-breaking scenario. In presence of magnetic impurities one can use the relation [26,27]:

$$\varepsilon_0(T) = \Delta[1 - (\Gamma/\Delta)^{2/3}]^{3/2} \qquad (17)$$

and, therefore, $\varepsilon_0 < \Delta$. Using the dependence $\Delta(T)$ (Fig. 2) and equations (3, 17), one can obtain the dependence $\varepsilon_0(T)$ and the value $\varepsilon_0(0)/T_c$. Using the corresponding parameters for $LuNi_2B_2C$, we obtain $2\varepsilon_0(0)/T_c = 4.4$. It is very essential that the order parameter Δ and, especially, the energy gap ε_0 strongly depend on temperature, even in the low temperature region. Indeed, the dependence $\varepsilon_0(T)$ is affected by the temperature dependence of Δ (see Fig. 2) and by the dependence $\Gamma(T)$ (see Eq. (3)). As a result, the comparison with experimental data should be carried out with considerable care. For example, the measurements in [13] were performed at $T = 4.2$ K. With use of equations (3, 7) and Figure 2, one can obtain $\varepsilon_0(4.2\text{ K}) = 3.2\,T_c$, in a very good agreement with the data [13], see also [30], and the review [12]. It would be interesting to perform the measurements of the energy gap ε_0 for various temperatures in the low temperature region, since the dependence $\varepsilon_0(T)$ is different from that for conventional superconductors.

Based on equation (7), one can calculate the dependence $\lambda_L(T)$. Unlike the critical field (see above and Fig. 1), it turns out that the dependence $\lambda_L(T)$ for $LuNi_2B_2C$ does not differ noticeable from the usual BCS dependence (except a small linear slope as $T \to 0$ K). It is interesting to note that the paper [11] contains a high quality data on λ_L for YNi_2B_2C and the measurements show a good agreement with the BCS function $\lambda_L(T)$. The calculation of the penetration depth for YNi_2B_2C based on our model will be presented elsewhere.

Figure 3 contains the calculated parameter $k(T)$ which displays a strong temperature dependence.

It is important to emphasize that the calculation of $\Delta(T)$, $\lambda_L(T)$, and $\kappa(T)$ invokes no additional parameters, but only values obtained from the calculation of $H_{c2}(T)$ above.

Let us finally discuss the dependence $H_{c1}(T)$. The measurements were carried out for $\mathbf{H} \parallel c$. The calculation is based on equations (7, 12, 16), and the results are presented in Figure 4. One should note that we analyze

the experimental data in the region $5\,\mathrm{K} < T < T_c$, where the term γ_2 (see Eq. (16)) is small. The function $\gamma_2(T)$, probably, increases up to $\gamma_2 \cong 1$ as $T \to 0$. The corresponding analysis will be described elsewhere. As for the intermediate temperatures and the region near T_c, one can see a good agreement between the theory (Eqs. (7, 12, 16)) and the data [15].

4 Conclusion

In this paper we focus on various properties of a new and interesting family of superconducting materials, borocarbides. The main results can be summarized as follows:

i) Based on the method of integrated Greens function, one can derive an expression for the upper critical field in the presence of magnetic scattering (Eq. (2)). Interaction between the magnetic moments makes the magnetic amplitude temperature dependent.
ii) We derived the expressions describing the full-range temperature dependencies of the order parameter $\Delta(T, \Gamma)$, the penetration depth (Eqs. (7, 10, 12)), the coherence length $\xi(T)$ (Eq. (9)), and the GL parameter $\kappa(T)$ (Eqs. (7, 9) and Figs. 2 and 3). The form of these dependencies in borocarbides is very different from that in conventional superconductors.
iii) A general expression describing the low critical field H_{c1} has been obtained (Eq. (16)).
iv) A detailed comparison with the data [15] on H_{c2} and H_{c1} for $LuNi_2B_2C$ has been carried out. The theory is in a very good agreement with the experimental data (Figs. 1 and 4).

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Appendix

i) In order to modify equation (8), it is useful to use the substitution: $\omega = \Delta \operatorname{tg}\theta - \Gamma \sin\theta$. Then we arrive at equation (9). Note that near T_c, $\theta = (\pi/2) - \Delta(\omega+\Gamma)^{-1}$. Then we obtain the relation

$$2\pi T \sum_{\omega>0} (\omega+\Gamma)^{-2}[Z - \Delta\omega(\omega+\Gamma)^{-2}] = 0 \qquad (A.1)$$

which leads to equation (13).

ii) Let us calculate the quantity δG. Based on equations (12, 14'), we obtain

$$\begin{aligned}\delta G = {} & 4\pi^2 \nu D_\perp \int_R^\infty \mathrm{d}\rho\, \rho K_1^2(\rho/\lambda)\lambda_L^{-2} T \\ & \times \sum_{\omega>0} \int_0^{|\Delta|} \mathrm{d}|\Delta| \sin^2\theta S^{-1} \\ & + 0.25 \int_0^\infty \mathrm{d}\rho\, \rho^{-1}(\partial/\partial\rho(\rho Q))^2 + \tilde{\varepsilon}(R) \\ S = {} & |\Delta| \cos^{-2}\theta - \Gamma\cos\theta \end{aligned} \qquad (A.2)$$

where $\xi \ll R \ll \lambda_L$.

The energy $\tilde{\varepsilon}(R)$ is equal to:

$$\tilde{\varepsilon}(R) = 2\pi\nu \int_0^R \mathrm{d}\rho\, \rho \int \mathrm{d}T_c^0/T_c^0(|\Delta_\infty^2|^2 - |\Delta(\rho)|^2). \quad (A.3)$$

Here Δ_∞ is the order parameter in the absence of the vortex and magnetic scattering. The energy $\tilde{\varepsilon}(R)$ can be written in the form:

$$\tilde{\varepsilon}(R) = (16e^2\lambda^2)^{-1}[\ln(R/\xi) + \gamma_1] \qquad (A.4)$$

where $\gamma \leq 1$. Evaluating the integral (A.2) (with use of substitution $S\mathrm{d}\theta = -\operatorname{tg}\theta \mathrm{d}|\Delta|$, we arrive at equation (16).

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